

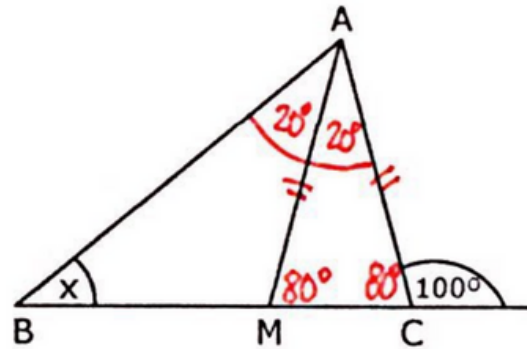
Solucionario

Triángulos

1.

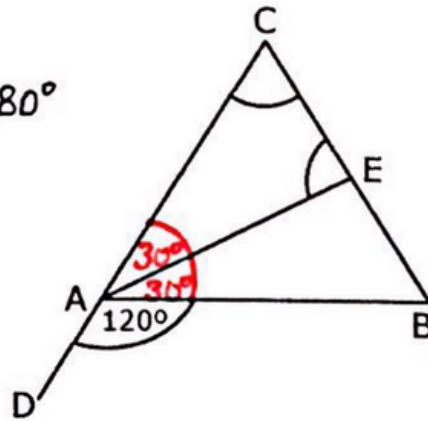
$$80^\circ = 20^\circ + x$$

$$x = 60^\circ$$



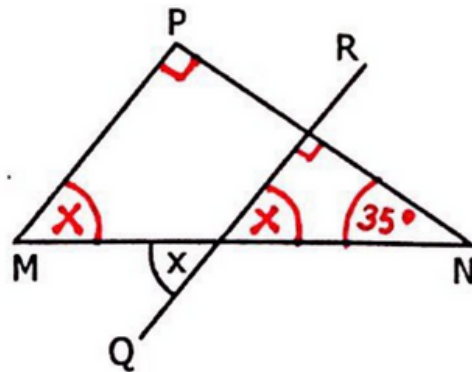
2. $\angle AEC + \angle ACE + 30^\circ = 180^\circ$

$$\angle AEC + \angle ACE = 150^\circ$$



3. $x + 90^\circ + 35^\circ = 180^\circ$

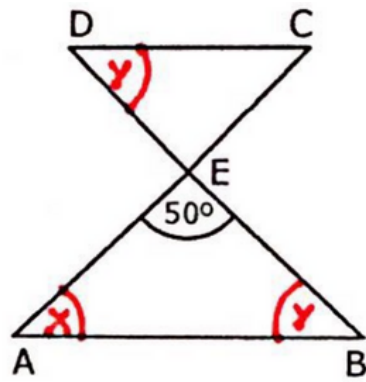
$$x = 55^\circ$$



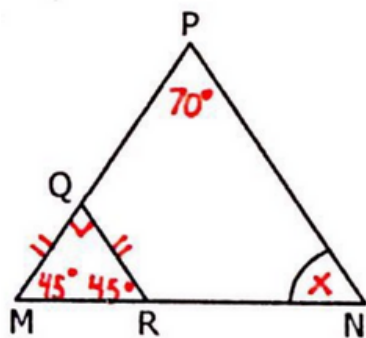
4.

$$x + y + 50^\circ = 180^\circ$$

$$x + y = 130^\circ$$



5.



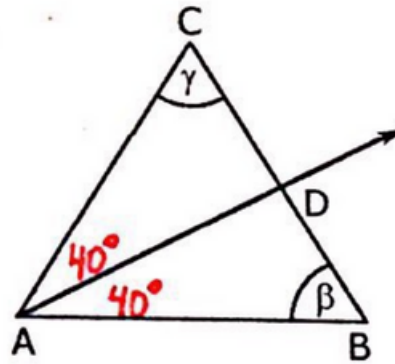
$$x + 70^\circ + 45^\circ = 180^\circ$$

$$x = 65^\circ$$

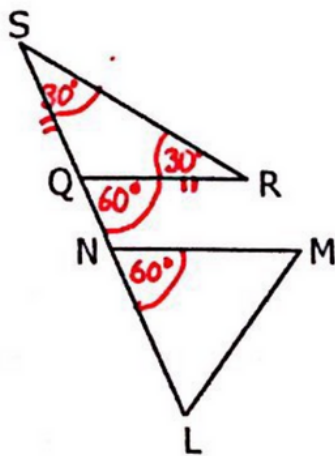
6. $\gamma + \beta + 80^\circ = 180^\circ$

$$\gamma + 80^\circ + 80^\circ = 180^\circ$$

$$\gamma = 20^\circ$$

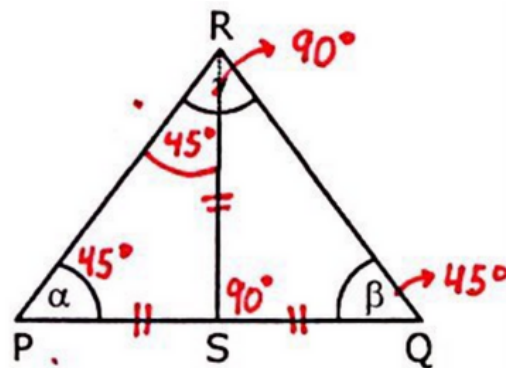


7.

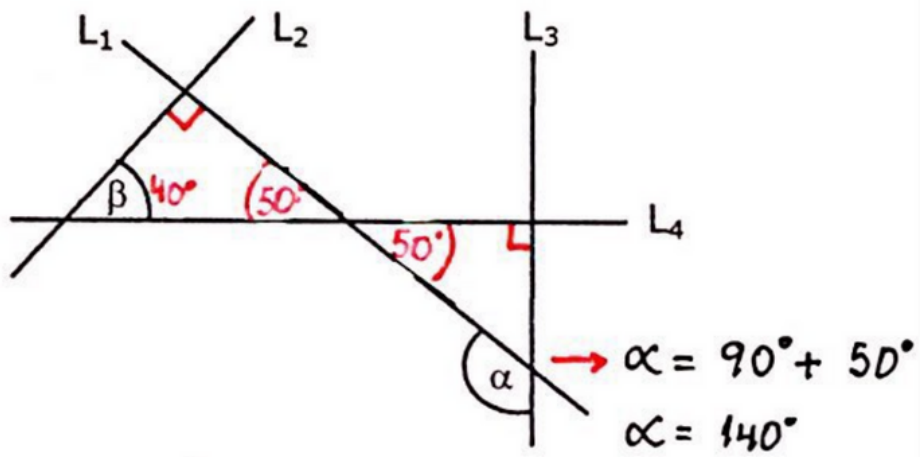


8. $\alpha = \gamma - \beta$

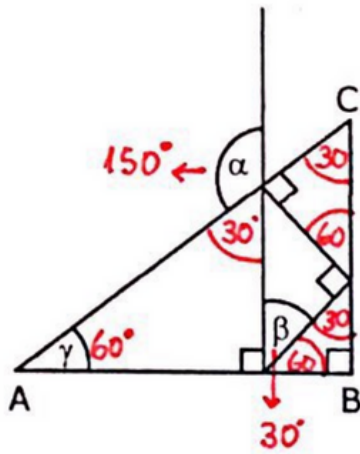
$$45^\circ = 90^\circ - 45^\circ$$



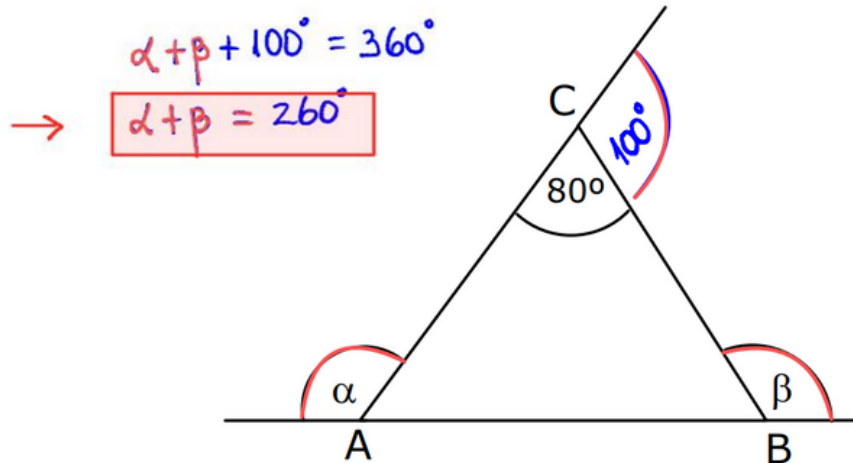
9.



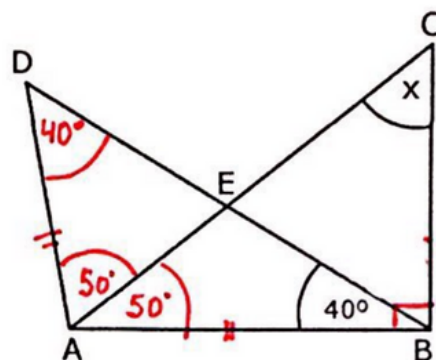
10.



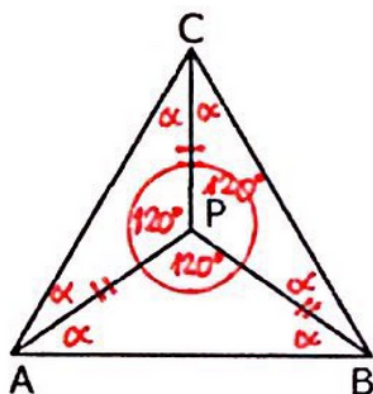
11.



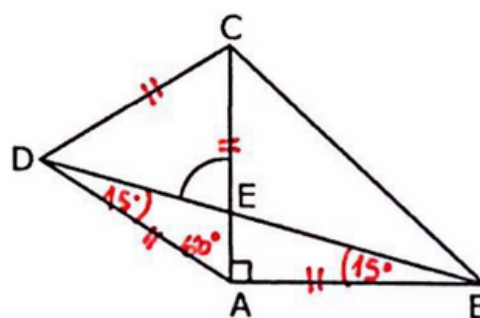
12. $x + 90^\circ + 50^\circ = 180^\circ$
 $x = 40^\circ$



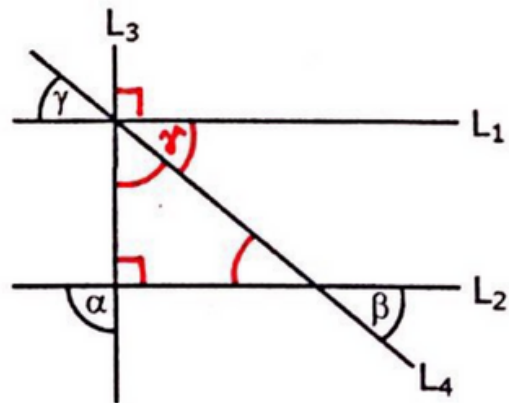
13.



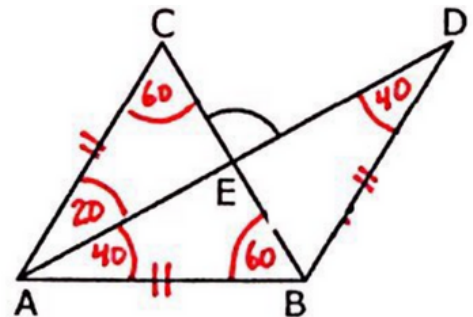
14. $\angle DAB = 60^\circ + 90^\circ = 150^\circ$
 $\Rightarrow \angle ABE = \angle ADE = 15^\circ$
 $\angle DEC = \angle ADE + \angle DAE$
 $\angle DEC = 15^\circ + 60^\circ = 75^\circ$



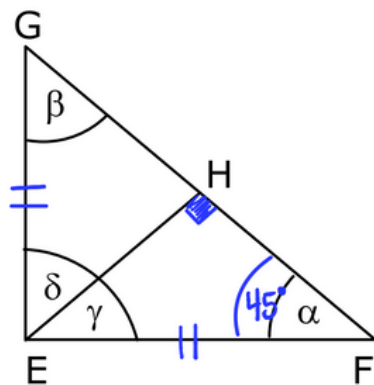
15. $\alpha = 90^\circ \Rightarrow \beta = 45^\circ$
 $\Rightarrow \gamma = 45^\circ$



16. $\angle CED = 20^\circ + 60^\circ = 80^\circ$



17.



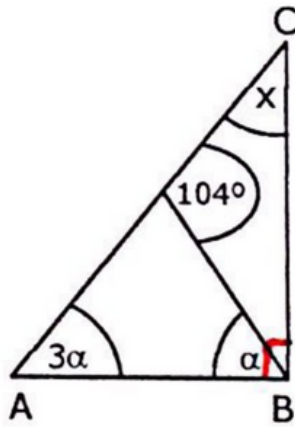
En $\triangle EHF$: $\gamma + 45^\circ + 90^\circ = 180^\circ \rightarrow \boxed{\gamma = 45^\circ}$

En $\triangle EFG$: $EF = EG \rightarrow \alpha = \beta = 45^\circ$, luego $\angle GEF = 90^\circ$

$\delta = 90^\circ - \gamma = 90^\circ - 45^\circ = 45^\circ$

luego I), II) y III) son verdaderas.

18.



$$104^\circ = \alpha + 3\alpha \Rightarrow \alpha = 26^\circ$$

$$x + 90^\circ + 3\alpha = 180^\circ$$

$$x + 90^\circ + 78^\circ = 180^\circ$$

$$x = 12^\circ$$

19.

$$\beta + 2\alpha = 180^\circ$$

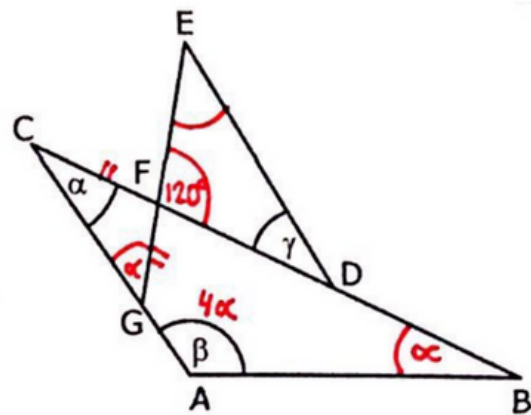
$$4\alpha + 2\alpha = 180^\circ$$

$$\alpha = 30^\circ$$

$$\gamma = \frac{\alpha}{2} = 15^\circ$$

$$\angle FED = 180^\circ - 120^\circ - 15^\circ$$

$$\angle FED = 45^\circ$$



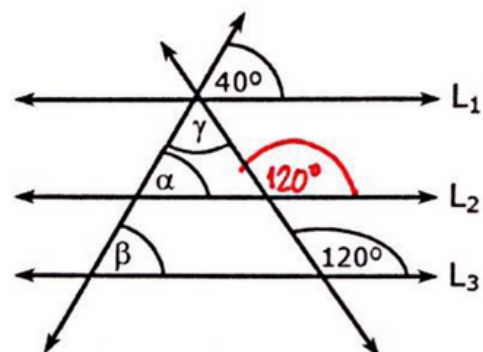
20.

$$120^\circ = \alpha + \gamma$$

$$\alpha = \beta = 40^\circ$$

$$\gamma = 120^\circ - \alpha = 120^\circ - 40^\circ = 80^\circ$$

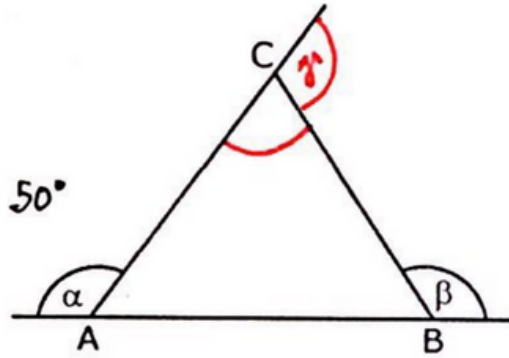
$$\alpha + \beta + \gamma = 160^\circ$$



21. $\alpha + \beta + \gamma = 360^\circ$

$$230^\circ + \gamma = 360^\circ$$

$$\gamma = 130^\circ \Rightarrow \angle ACB = 50^\circ$$



22. $3[90^\circ - (\alpha - 10^\circ)] = 180^\circ - (\alpha - 20^\circ)$

$$3[90^\circ - \alpha + 10^\circ] = 180^\circ - \alpha + 20^\circ$$

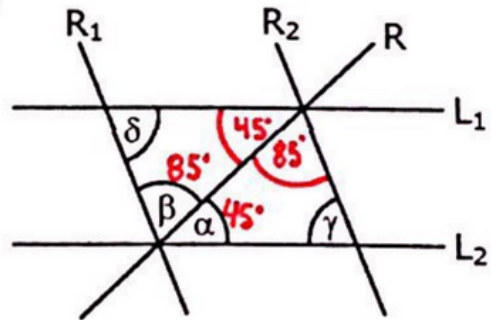
$$3[100^\circ - \alpha] = 200^\circ - \alpha$$

$$300^\circ - 3\alpha = 200^\circ - \alpha$$

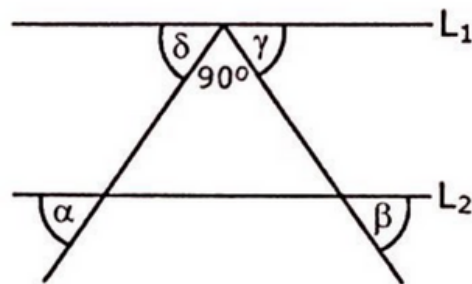
$$100^\circ = 2\alpha \Rightarrow \alpha = 50^\circ$$

$$\text{Complemento de } \alpha = 90^\circ - \alpha = 90^\circ - 50^\circ = 40^\circ$$

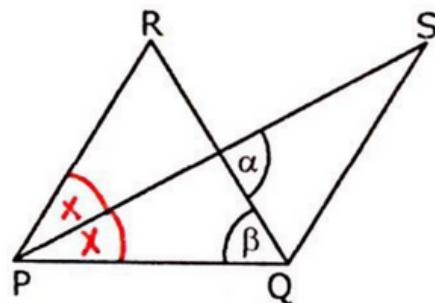
23. $\gamma + 45^\circ + 85^\circ = 180^\circ \Rightarrow \gamma = 50^\circ$
 $\delta + 45^\circ + 85^\circ = 180^\circ \Rightarrow \delta = 50^\circ$
 $\gamma + \delta = 100^\circ$



24. Si $\alpha = \gamma$ como $\gamma = \beta$
 entonces $\alpha = \beta + \gamma$
 Además $\alpha = \delta$



25. $\angle QPR = 2x$
 $\alpha = \beta + x \Rightarrow 60^\circ = 30^\circ + x$
 $\Rightarrow x = 30^\circ$
 $\angle QPR = 2x = 60^\circ$

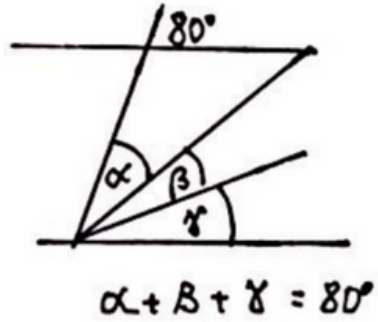


26. Con (2) por sí sola se determina
 que $125^\circ = \angle ACD + 90^\circ$

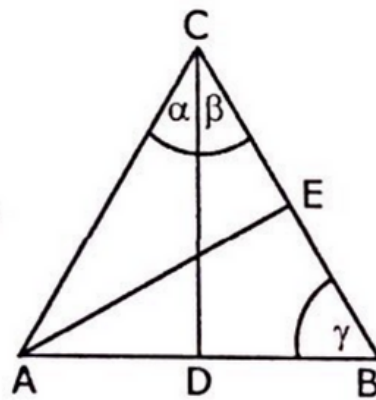
27. Con (2) por sí sola
si $\beta = \gamma$ entonces $L_1 \parallel L_2$

28. Con (1) y (2) juntas
se determina que los tres
ángulos interiores son iguales

29. Con (2) por sí sola y arreglando la figura se tiene



30. Con (1) por sí sola se determina que
 $\angle CAB = \angle ABC = \angle BCA$



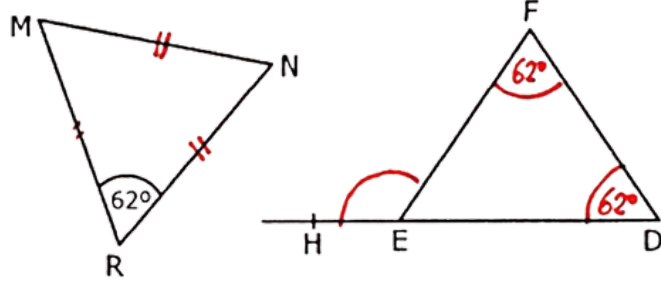
Solucionario Congruencia

1.

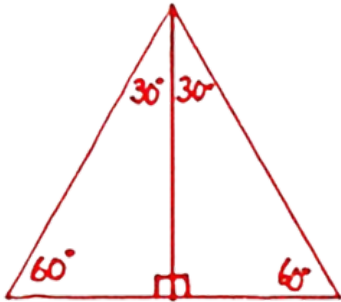
$$\triangle MRN \cong \triangle DFE.$$

$\downarrow$ $\downarrow$
 62° 62°

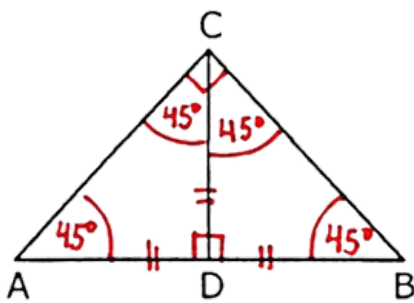
Si $\overline{MN} \cong \overline{NR} \Rightarrow$
 $\angle MRN = \angle NMR = 62^\circ$
 $\therefore \angle EDF = 124^\circ$



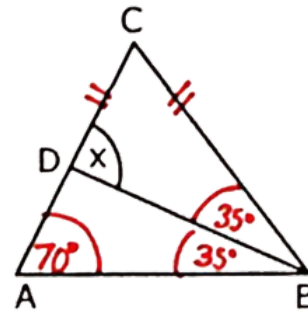
2.



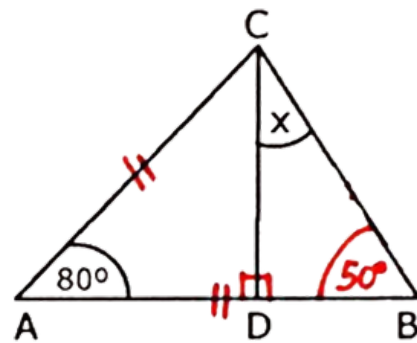
3.



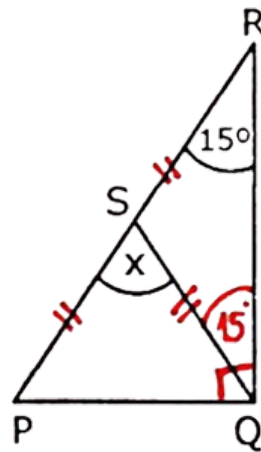
4. $\angle X = 70^\circ + 35^\circ = 105^\circ$



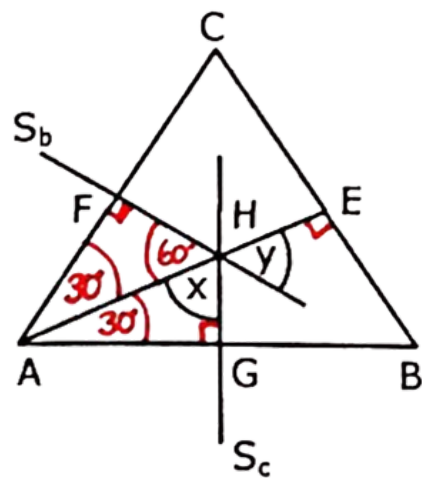
5. $X + 90^\circ + 50^\circ = 180^\circ$
 $X = 40^\circ$



6. $\angle x = 15^\circ + 15^\circ = 30^\circ$



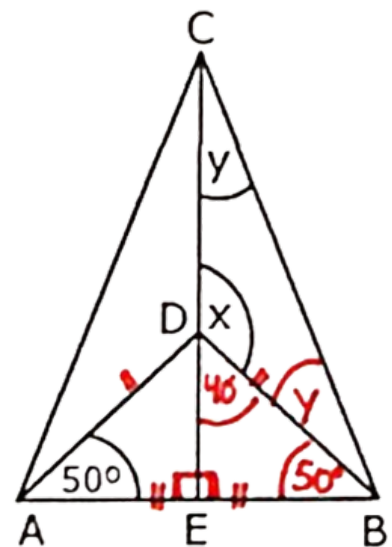
7. $\angle x + \angle y = 60^\circ + 60^\circ = 120^\circ$



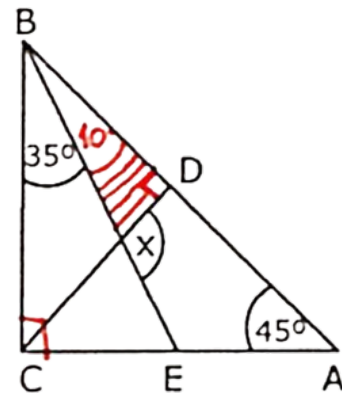
8. $\angle x + 40^\circ = 180^\circ \Rightarrow \angle x = 140^\circ$

$40^\circ = 2\angle y \Rightarrow \angle y = 20^\circ$

$\angle x + \angle y = 160^\circ$



9. $X = 90^\circ + 10^\circ = 100^\circ$



10.

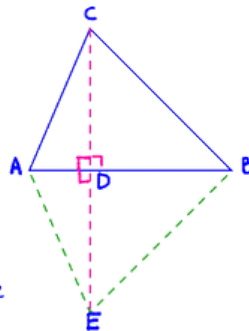
- I) $\triangle ABC \cong \triangle ABE$ ✓
- II) $\triangle ADC \cong \triangle ADE$ ✓
- III) $\triangle ADE \cong \triangle BDC$ F

II) Verdadero. $\triangle ADC \cong \triangle ADE$
por L-A-L

$$\overline{AD} - l - \overline{CD}$$

$$\overline{AD} - l - \overline{DE}$$

III) Falso. Se cumple solamente
si $\overline{AD} \cong \overline{DB}$



I) Verdadero. Como $\overline{CD} \cong \overline{DE}$, entonces:

$\triangle ADC \cong \triangle ADE$ por L-A-L

$\overline{AD} - l - \overline{CD}$ y $\overline{AD} - l - \overline{DE}$

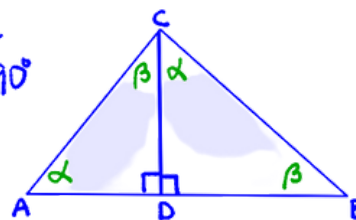
$\triangle BDC \cong \triangle BDE$

$\overline{BD} - l - \overline{CD}$ y $\overline{BD} - l - \overline{DE}$

Entonces: $\triangle ABC \cong \triangle ABE$

11.

C) No permite
si $\alpha + \beta = 90^\circ$



$$\triangle BDC \not\cong \triangle ADC$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \beta & \downarrow & \alpha \\ & l & \end{array}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \alpha & \downarrow & \beta \\ & l & \end{array}$$

NO se cumple el
orden de la congruencia

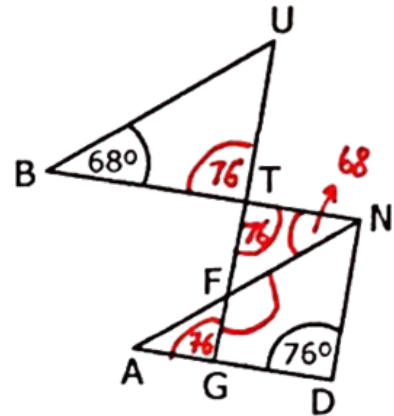
12.

$\angle UBT = \angle FNT = 68^\circ$
(alternos internos entre paralelas)

$\angle BTU = \angle AGF$

(Correspondientes entre paralelas)

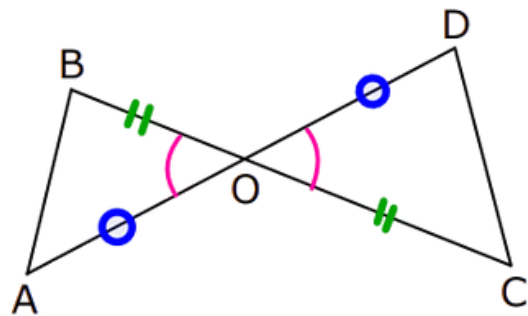
$\angle GFN = 68^\circ + 76^\circ = 144^\circ$



13.

A), B), C) y D) son
insuficientes. No es posible
probar los criterios LAL,
ALA ni LLL.

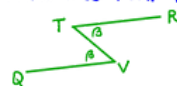
E) suficiente
Se cumple criterio
L-A-L (ver figura)



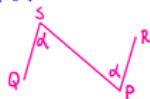
14.

- I) $\overline{TR} \parallel \overline{VQ}$
- II) $\overline{PR} \parallel \overline{SQ}$
- III) $\angle SQV \cong \angle PRT$

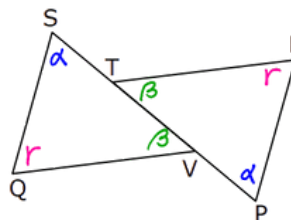
I) Verdadero.
Como $\angle PTR \cong \angle SVQ$
Entonces $\overline{TR} \parallel \overline{VQ}$



II) Verdadero.
Como $\angle PSQ \cong \angle SPR$
Entonces $\overline{PR} \parallel \overline{SQ}$



III) Verdadero
por la congruencia
entre el $\triangle PTR$ y el $\triangle SVQ$
 $\angle SQV \cong \angle PRT$

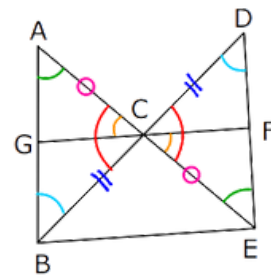


15.

- I) $\overline{GC} \cong \overline{FC}$ ✓
 II) $\angle BAC \cong \angle DEC$ ✓
 III) $\overline{GC} \perp \overline{AB}$ y $\overline{CF} \perp \overline{DE}$ F

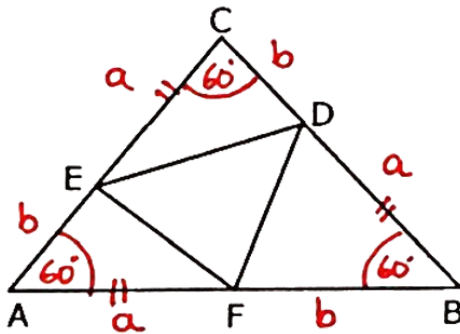
I) Verdadero.
 $\triangle ACG \cong \triangle ECF$
 por ALA.
 Entonces $\overline{GC} \cong \overline{FC}$

II) Verdadero.
 $\triangle ABC \cong \triangle EDC$
 $\Rightarrow \angle BAC \cong \angle DEC$

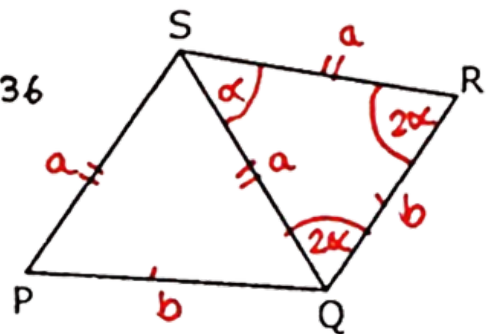


III) Falso.
 No necesariamente
 ocurre esto

16.

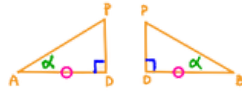


17. De la figura, $5\alpha = 180^\circ \Rightarrow \alpha = 36$
 $\angle SPD = 2\alpha = 72^\circ$

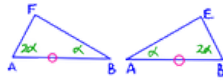


18.

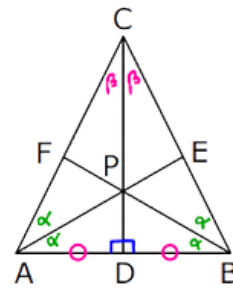
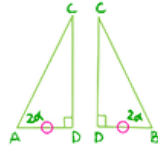
A) Verdadero.
por LAL



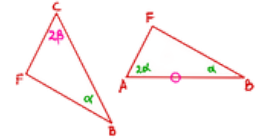
B) Verdadero
por ALA



C) Verdadero
por ALA



D) Falso.
No se puede
demostrar
criterio LAL,
ALA o LLL.



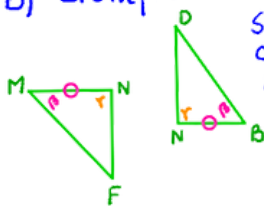
19.

A) No siempre

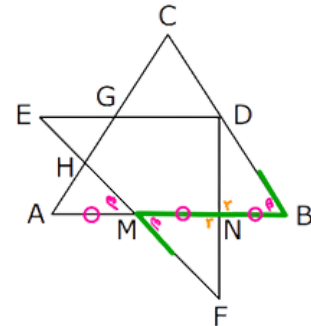


No se puede
demostrar LAL
ni ALA

B) Siempre



Se cumple
criterio
ALA



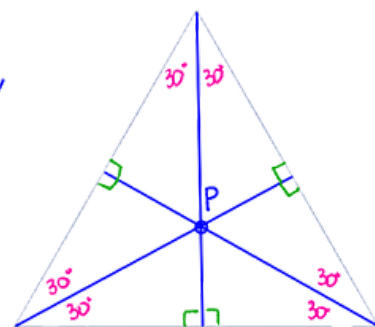
20.

- I) Es vértice de seis triángulos congruentes. ✓
- II) Es vértice de seis triángulos equiláteros. F
- III) Es vértice de seis triángulos de igual perímetro ✓

I) Son congruentes por ALA, LAL y LLL

II) Falso. Todos son Δ s rectángulos.

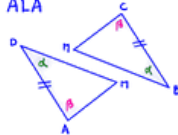
III) Verdadero. A) ser congruentes
tienen igual perímetro.



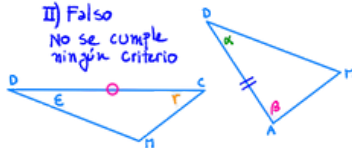
21.

- I) $\triangle AMD \cong \triangle CMB$ ✓
 II) $\triangle AMD \cong \triangle CMD$ ✗
 III) $\triangle CDA \cong \triangle ABC$ ✓

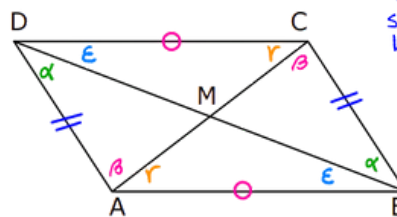
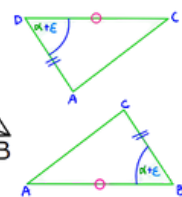
I) Verdadero
 se cumple criterio
 ALA



II) Falso
 No se cumple
 ningún criterio



III) Verdadero.
 se cumple criterio
 LAL



22.

$$CD = 3k$$

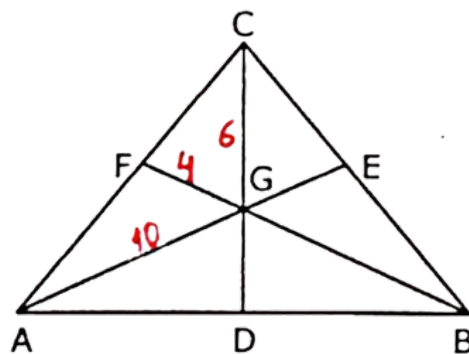
$$CD = 9$$

$$AE = 5k = 15 \Rightarrow k = 3$$

$$BF = 4k$$

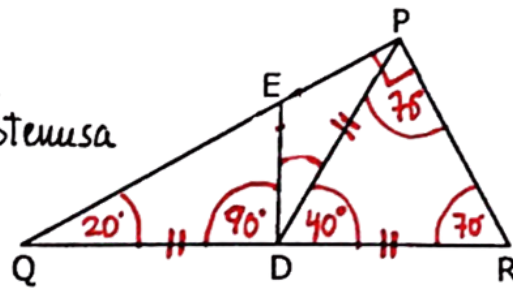
$$BF = 12$$

$$CG + AG + GF = 6 + 10 + 4 = 20$$



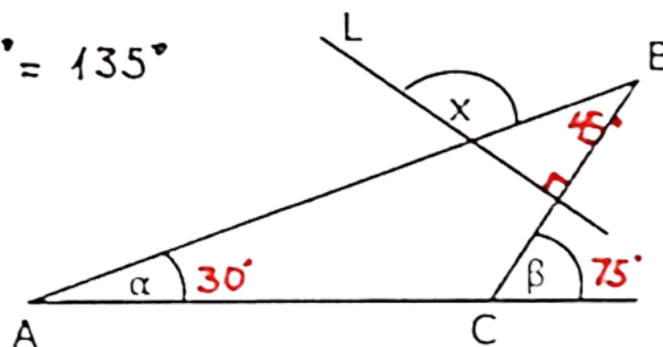
23. Por propiedad de la transversal de gravedad relativa a la hipotenusa

$$QD = PD = DR$$

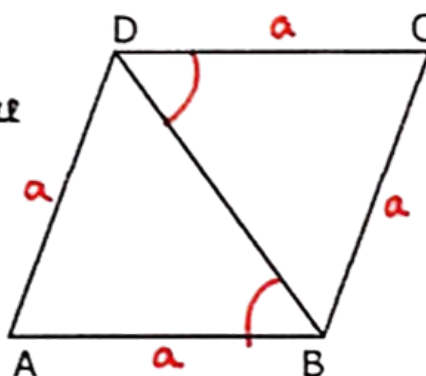


24. $120^\circ + \delta = 180^\circ \Rightarrow \delta = 60^\circ$

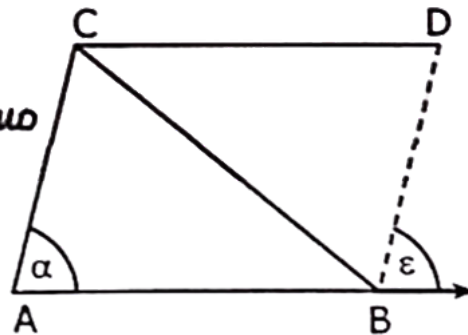
25. $\lambda = 45^\circ + 90^\circ = 135^\circ$



26. Con (1) se determina que $\angle ABD = \angle CDB$ porque son alternos internos entre paralelas

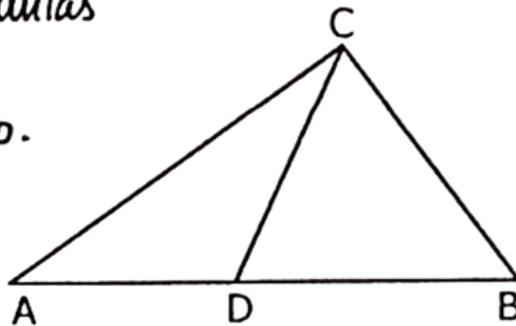


27. Con (1) y con (2) por separado se determina que ABCD es un paralelogramo

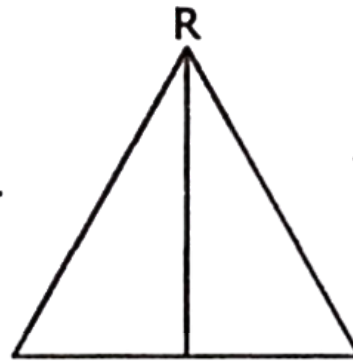


28. Con (1) y con (2) juntas se determina que el Δ es rectángulo.

$$AD = CD = DB$$



29. Con (1) y con (2) juntas
se determina que
 $\triangle PSR \cong \triangle QSR$ y por
lo tanto el $\triangle PQR$ es
isósceles.



30. (1) Insuficiente
No se indica la base
del $\triangle$ isósceles.
- (2) Suficiente
Se cumple criterio
ALA

